

Solⁿ: Let (x', y') be the coordinates of its centre, then we have

$$17x' - 6y' + 23 = 0$$

$$\text{and } -6x' + 8y' - 14 = 0$$

Solving above equations, we get $x' = -1$ and $y' = 1$

Therefore, coordinates of the centre are $(-1, 1)$

Also,

$$\begin{aligned} c' &= gx' + fy' + c \\ &= -23 - 14 + 17 \\ &= -20 \end{aligned}$$

Hence, equation of the conic, referred to origin at centre and axes being parallel to original axes, will be

$$17x'^2 - 12x'y' + 8y'^2 + c' = 0$$

$$\Rightarrow 17x'^2 - 12x'y' + 8y'^2 - 20 = 0$$

$$\Rightarrow \frac{1}{20} (17x'^2 - 12x'y' + 8y'^2) = 1$$

Hence, the length of its axes will be

given by,

$$\left(\frac{17}{20} - \frac{1}{r^2}\right) \left(\frac{8}{20} - \frac{1}{r^2}\right) = \left(\frac{-6}{20}\right)^2$$

$$\Rightarrow (17r^2 - 20) (2r^2 - 5) = 9$$

of the conic (1) as,

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c - \frac{\Delta}{ab-h^2} = 0$$

Tracing of conics:

In case, the conic be an ellipse or a hyperbola, then find out its

- (i) Centre,
- (ii) Equation of the conic referred to centre as origin.
- (iii) lengths and positions of the axes,
- (iv) Eccentricity, foci, directrices,
- (v) points where the conic meets the co-ordinate axes,
- (vi) Equation of asymptotes in case of hyperbola, and
- (vii) Rough sketch of the conic

This will be called the tracing of the conic.

Solved example:

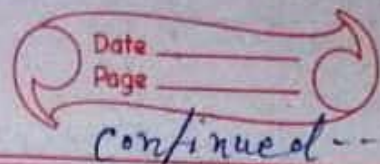
(1) Trace the conic,

$$17x^2 - 12xy + 8y^2 + 46x - 28y + 7 = 0$$

Also find the equation of its directrices.

Analytical Geometry of two dimension

By Awadh Bihari Yadav



Equation of Asymptotes.

Let $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
be the equation of the conic, — (1)

then the equation of its asymptotes
will be

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c + \lambda = 0 \quad (2)$$

since asymptotes are straight
line, hence (2) will represent a pair of
straight lines. Therefore, applying the
condition that a second degree equation
represent a pair of straight lines,

we get

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c + \lambda \end{vmatrix} = 0$$

which gives

$$\lambda(ab - h^2) + \Delta = 0 \quad (3)$$

where $\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$

putting the value of λ from (3) in (2)

we get the equation of the asymptotes

$$\Rightarrow 25x^4 - 125x^2 + 100 = 0$$

which is quadratic in x^2 . Hence solving it we get $x^2 = 4$ or 1 .

since, both the value of x^2 are positive the conic is an ellipse.

its semi-axes are of lengths $R=2$ and $r=1$ and the gradient of major axes is equal to

$$-\left(a - \frac{1}{R^2}\right) / h = -\left(\frac{17}{20} - \frac{1}{4}\right) / \frac{-3}{10} = 2$$

where the ellipse meet the axis of x , we have

$$17x^2 + 46x + 17 = 0$$

which gives

$$x = \frac{-23}{17} \pm \frac{4}{17} \sqrt{15}$$

$$= -0.44 \text{ and } -2.26 \text{ nearly}$$

Also, where it meet the axis of y , we

$$\text{get } 8y^2 - 28y + 7 = 0$$

which on solving gives,

$$y = 2.72 \text{ and } .78 \text{ nearly}$$