Sol^n: Let (x', y') be the coordinates of its centre, then we have

$$17x' - 6y' + 23 = 0$$

and $$-6x' + 8y' - 14 = 0$$

Solving above equations, we get

$$x' = -1 \text{ and } y' = 1$$

Therefore, coordinates of the centre are $(-1, 1)$

Also,

$$c' = gx' + fy' + c$$
$$= -23 - 14 + 17$$
$$= -20$$

Hence, equation of the conic, referred to origin at centre and axes being parallel to original axes, will be

$$17x^2 - 12xy + 8y^2 + c' = 0$$
$$\Rightarrow 17x^2 - 12xy + 8y^2 - 20 = 0$$
$$\Rightarrow \frac{1}{20}(17x^2 - 12xy + 8y^2) = 1$$

Hence, the length of its axes will be given by,

$$\left(\frac{17}{20} - \frac{1}{r^2}\right)\left(\frac{8}{20} - \frac{1}{r^2}\right) = \left(\frac{-6}{20}\right)^2$$
$$\Rightarrow (17r^2 - 20)(2r^2 - 5) = 9$$

of the conic (1) as.

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c - \frac{\Delta}{ab-h^2} = 0$$

Tracing of conics :

In case, the conic be an ellipse or a hyperbola, then find out its:

(i) centre,

(ii) Equation of the conic referred to centre as origin.

(iii) lengths and positions of the axes,

(iv) Eccentricity, foci, directrices,

(v) points where the conic meets the co-ordinate axes,

(vi) Equation of asymptotes in case of hyperbola, and

(viii) Rough sketch of the conic

This will be called the tracing of the conic.

Solved example:

(1) Trace the conic,

$$17x^2 - 12xy + 8y^2 + 46x - 28y + 7 = 0$$

Also find the equation of its directrices.

Analytical Geometry of two dimension

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continued...

Equation of Asymptotes :

let $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ — ①

be the equation of the conic,

then the equation of its asymptotes will be

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c + \lambda = 0 \quad ②$$

since asymptotes are straight line, hence ② will represent a pair of straight lines. therefore, applying the condition that a second degree equation represent a pair of straight lines,

we get

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c+\lambda \end{vmatrix} = 0$$

which gives

$$\lambda(ab - h^2) + \Delta = 0 \quad ③$$

$$\text{where } \Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

putting the value of λ from (3) in ②

the equation of the asymptotes

$\Rightarrow 25r^4 - 125r^2 + 100 = 0$

which is quadratic in r^2. Hence solving it we get $r^2 = 4$ or 1.

since, both the value of r^2 are positive the conic is an ellipse.

its semi-axes are of lengths $R = 2$ and $r = 1$ and the gradient of major axes is equal to

$$-\left(a - \frac{1}{R^2}\right)\Big/h = -\left(\frac{17}{20} - \frac{1}{4}\right)\Big/ {-3/10}$$
$$= 2.$$

where the ellipse meet the axis of x, we have

$$17x^2 + 46x + 17 = 0$$

which gives

$$x = \frac{-23}{17} \pm \frac{4}{17}\sqrt{15}$$

$$= -.44 \text{ and } -2.26 \text{ nearly}$$

Also, where it meet the axis of y, we get $8y^2 - 28y + 7 = 0$

which on solving gives,

$y = 2.72$ and $.78$ nearly.